

1. Consider the following series and determine if they converge or diverge. In case of an alternating series specify if the convergence is absolute or conditional.

(a)  $\sum_{n=1}^{\infty} \frac{2^n (n!)}{n^n}$

(b)  $\sum_{n=1}^{\infty} (-1)^n \frac{\ln(n)}{n^2}$

(c)  $\sum_{n=1}^{\infty} (-1)^n \tan\left(\frac{1}{n}\right)$

(d)  $\sum_{n=1}^{\infty} \frac{\sqrt[n]{n}}{n^2}$

(e)  $\sum_{n=1}^{\infty} \frac{\sqrt{n}(n^2 + n + 1)}{n^3 + 1}$

(f)  $\sum_{n=1}^{\infty} \frac{n \ln n}{2^n}$

(g)  $\sum_{n=2}^{\infty} (-1)^n \frac{1}{n(\ln n)^2}$

2. Give examples of two series  $\sum_{n=1}^{\infty} a_n$  and  $\sum_{n=1}^{\infty} b_n$  that both converge but such that the series  $\sum_{n=1}^{\infty} a_n b_n$  diverges.

3. For what values of  $x$  does the power series  $\sum \frac{(4x - 5)^n}{n^{3/2}}$  converge? Explain

4. (a) Find the McLaurin series for  $f(x) = \ln(1 + x)$  and determine for what values of  $x$  the series converges.

(b) Deduce  $\lim_{x \rightarrow 0} \frac{\ln(1 + x)}{x}$

- (c) Using the McLaurin polynomial of degree 4, find an approximation for  $\ln(2)$

5. Find the Taylor series for  $f(x) = x^3 - 2x^2 + x + 1$  with  $a = 1$

6. (a) Using the series for  $e^x$ , find the series for  $e^{x^2}$ .

- (b) Using the result in part a, express the integral  $\int e^{x^2} dx$  as an infinite series.